

Perturbation Methods
Extra Credit
Due May 11

The Poisson-Nernst-Planck model for flow of ions through a membrane consists of the following equations: for $0 < x < 1$,

$$\begin{aligned}\frac{dp}{dx} + p\frac{d\phi}{dx} &= -\alpha, \\ \frac{dn}{dx} - n\frac{d\phi}{dx} &= -\beta, \\ \varepsilon^2\frac{d^2\phi}{dx^2} &= -p + n.\end{aligned}$$

The boundary conditions are $\phi(0) = 1$, $\phi(1) = 0$, $p(0) = 4$, and $n(0) = 1$. In these equations, p and n are the concentrations of the ions with valency 1 and -1, respectively, and ϕ is the potential. Assume that α and β are positive constants that satisfy $\kappa < 1$, where

$$\kappa = \frac{\alpha + \beta}{2\sqrt{p(0)n(0)}}.$$

Also, you can assume that $\alpha \neq \beta$. Find a first-term approximation for $p(1)$ (your answer should be an explicit function of α and β).