

HW 6

Due Dec 3

1. In the following, in addition to writing down the partial differential equations, you must provide an explanation of where they come from.

a) text 1(a), pg 477

b) text 1(b), pg 477

c) Four species move randomly along the x -axis, and react according to the reaction below.

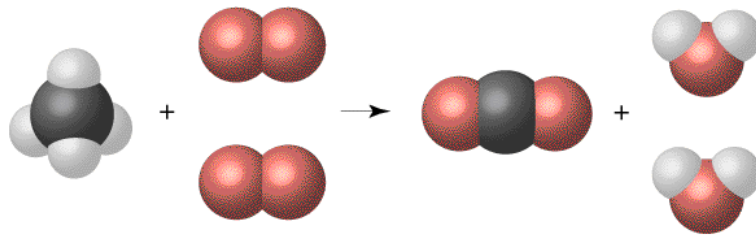


Figure 1: Reaction for Exercise 1(c).

2. A model for bacteria that move into regions with a food source is

$$\frac{\partial b}{\partial t} = \frac{\partial}{\partial x} \left(D \frac{\partial b}{\partial x} - \frac{\alpha b}{n} \frac{\partial n}{\partial x} \right)$$

$$\frac{\partial n}{\partial t} = -kb$$

where $b(x, t)$ is the density of bacteria and $n(x, t)$ is the density of food (or nutrients). Identify the J and Q for each equation, and provide physical assumptions that give rise to these functions.

3. This problem concerns the following reaction-diffusion equation

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} + c(1 - c^2).$$

In this problem, c is a relative concentration, which means that it can be positive or negative.

a) Find the constant steady-states and determine their stability.

b) Suppose the initial-condition is $u(x, 0) = f(x)$, where

$$f(x) = \frac{1}{1 + e^x}.$$

Sketch this function and, using your results from part (a), describe what the solution of the reaction-diffusion problem looks like at $t = \infty$.

c) Redo part (b) using

$$f(x) = \frac{1 - e^x}{1 + e^x}.$$

d) Describe how this problem can be used to produce the coloring on a belted Galloway cow (also known as an Oreo cow).

4. Suppose in the Michaelis-Menten problem the four species move randomly along the x -axis.
- What are the resulting four reaction-diffusion equations?
 - Suppose the species are confined to a closed interval $a \leq x \leq b$, which means each species satisfies a no flux condition at the endpoints. Write down the corresponding boundary conditions for each species.
 - Assuming the species reach a uniform steady-state, find the value(s) for E and S .
5. The Gierer-Meinhardt model consists of the following two equations

$$\begin{aligned}\partial_t u &= D_1 \partial_x^2 u + \frac{u^2}{v} - bu \\ \partial_t v &= D_2 \partial_x^2 v + u^2 - v\end{aligned}$$

where b is a positive constant.

- Find the uniform steady-state (there is only one and you need to explain why).
- Assuming no diffusion (so, $D_1 = D_2 = 0$), what condition must be imposed on b so the steady-state is asymptotically stable?
- In addition to the condition from part (b), what two inequalities must be satisfied for there to be a Turing instability?
- Show that the inequalities in part (c) reduce to the requirement that $D_1 < (3 - 2\sqrt{2})bD_2$
- Which is the inhibitor and which is the activator? Make sure to explain how you arrive at your conclusion.