HW5 #1

Solutions to HW5

1. a) $\Delta x = 30d = 30 \cdot 3\times10^{-7} = 900\times10^{-7} = 9\times10^{-5}$ cm

$$D = \frac{\Delta x^2}{2\Delta t} \Rightarrow \Delta t = \frac{\Delta x^2}{2D} = \frac{81\times10^{-10}}{0.4} = \frac{8.1\times10^{-10}}{4\times10^{-1}} \approx 2\times10^{-9} \text{ sec}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{9\times10^{-5}}{2\times10^{-9}} = 4.5\times10^{4} \frac{\text{cm}}{\text{sec}}$$

$$\#\text{ of collisions} = \frac{1}{\Delta t} \approx \frac{1}{2\times10^{-9}\text{ sec}} = \frac{1}{2}\times10^{9} \frac{1}{\text{sec}}$$

b) $\Delta x = 30 \cdot 3\times10^{-8} = 9\times10^{-6}$ cm

$$\Delta t = \frac{81\times10^{-12}}{4\times10^{-5}} \approx 20\times10^{-7} = 2\times10^{-6} \text{ sec}$$

$$v = \frac{\Delta x}{\Delta t} \approx \frac{9\times10^{-6}}{2\times10^{-6}} = 4.5 \frac{\text{cm}}{\text{sec}}$$

$$\text{collisions} = \frac{1}{2\times10^{-6}} = \frac{1}{2}\times10^{6}/\text{sec}$$

2. $$\underset{\text{I}}{\longrightarrow}\; a \;\underset{\text{II}}{\longrightarrow}\; b \;\underset{\text{III}}{\longrightarrow} x$$

$\text{erfc}(\infty) = 0$
$\text{erfc}(-\infty) = 2$
$\text{erfc}(0) = 1$

I. $x<a \Rightarrow \lim_{t\downarrow 0} c = \frac{1}{2}\left[\text{erfc}(-\infty) - \text{erfc}(-\infty)\right] = 0$

II. $a<x<b \Rightarrow \lim_{t\downarrow 0} c = \frac{1}{2}\left[\text{erfc}(-\infty) - \text{erfc}(\infty)\right] = \frac{1}{2}(2-0) = 1$

III. $b<x \Rightarrow \lim_{t\downarrow 0} c = \frac{1}{2}\left[\text{erfc}(\infty) - \text{erfc}(\infty)\right] = 0$

$x=a$: $\lim c = \frac{1}{2}\left[\text{erfc}(-\infty) - \text{erfc}(0)\right] = \frac{1}{2}(2-1) = \frac{1}{2}$

$x=b$: $\lim c = \frac{1}{2}\left[\text{erfc}(0) - \text{erfc}(\infty)\right] = \frac{1}{2}[1-0] = \frac{1}{2}$

3. point-source solution $= \dfrac{1}{2\sqrt{\pi D t}}\, e^{-\frac{x^2}{4Dt}}$

a) & b) $f(t) = \dfrac{1}{2\sqrt{\pi D t}}\, e^{-x^2/4Dt}$

$$\Rightarrow f' = \frac{1}{2\sqrt{\pi D}}\left[-\frac{1}{2}t^{-3/2}e^{(\,)} + t^{-1/2}\cdot\frac{x^2}{4Dt^2}e^{(\,)}\right]$$

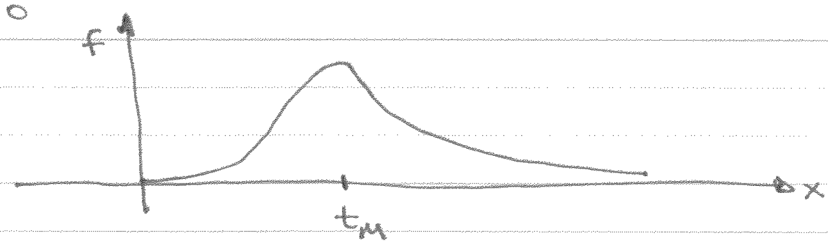
HW5 #2

$$f' = 0 \Rightarrow -\tfrac{1}{2} t^{-3/2} + t^{-1/2} \cdot \frac{x^2}{4Dt^2} = 0$$

$$x^2 = \tfrac{1}{2} t^{-3/2} \cdot 4D t^{5/2} = 2D \cdot t$$

$$\Rightarrow \quad t_M = \frac{x^2}{2D}$$

$f(0) = 0$ and $f(\infty) = 0$
$x \neq 0$



c) $D = 0.2 \Rightarrow t_M = \dfrac{(10^2\,cm)^2}{2 \cdot 0.2} = \dfrac{10^4}{0.4}$ sec =

$D = 2\times10^{-5} \Rightarrow t_M = \dfrac{(10^2\,cm)^2}{2\times 2\times10^{-5}} = \dfrac{10^4}{4\times10^{-5}} = \frac{1}{4}\times 10$ sec =

4. a) $\partial_t n = \dfrac{1}{2\sqrt{\pi d_0}} \left\{ -\tfrac{1}{2} t^{-3/2} e^{()} + t^{-1/2} \cdot e^{()} \left[+ \dfrac{2(\alpha - v_0 t)}{4 d_0 t} \cdot v_0 + \dfrac{(\alpha - v_0 t)^2}{4 d_0 t^2} \right] \right\}$

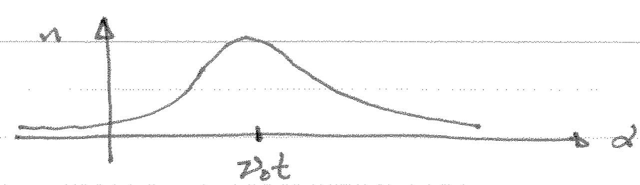
$\partial_\alpha n = \dfrac{1}{2\sqrt{\pi d_0}} \left\{ t^{-1/2} \cdot e^{()} \cdot \left[-2 \dfrac{(\alpha - v_0 t)}{4 d_0 t} \right] \right\}$

$\partial_\alpha^2 n = \dfrac{1}{2\sqrt{\pi d_0}} \left\{ t^{-1/2} e^{()} \cdot \left[-2 \dfrac{(\alpha - v_0 t)}{4 d_0 t} \right]^2 + t^{-\frac{1}{2}} e^{()} \left[\dfrac{-2}{4 d_0 t} \right] \right\}$

Now, plug into PDE and show they add up correctly.

b) max: $\partial_\alpha n = 0 \Rightarrow$ (from above) $\alpha = v_0 t$ $\quad n\big|_{\alpha = v_0 t} = \dfrac{1}{2\sqrt{\pi d_0 t}}$

$n\big|_{\alpha \to \infty} = 0 \qquad n\big|_{\alpha \to -\infty} = 0$



c) max location for n is moving to right (note the max value is same for both solutions) — otherwise they are same (symmetric about max, limits at infinity, max value, etc)