

## HW 5

Due Nov 12

1. According to the Einstein-Smoluchowski theory for the random motion of particles in a gas (or fluid), the spatial jump  $\Delta x$  is the average distance that the particle travels between collisions (what is known as the mean free path of the particle), and  $\Delta t$  is the average time between collisions. Also, the average speed of the particle is defined as  $v = \Delta x / \Delta t$ .

a) In air the mean free path is in the neighborhood of 30 times the average molecular separation distance  $d$ . In air suppose  $d$  is approximately  $3 \times 10^{-7}$  cm. Given that for air  $D \approx 0.2 \frac{\text{cm}^2}{\text{sec}}$ , approximately how long is it between collisions? Approximately how fast are the molecules traveling? How many collisions are there per second?

b) In water the mean free path is in the neighborhood of 30 times the average molecular separation distance  $d$ . In water suppose  $d$  is approximately  $3 \times 10^{-8}$  cm. Given that for water  $D \approx 2 \times 10^{-5} \frac{\text{cm}^2}{\text{sec}}$ , approximately how long is it between collisions? Approximately how fast are the molecules traveling? How many collisions are there per second?

2. Consider the following function

$$c(x, t) = \frac{1}{2} \left[ \operatorname{erfc} \left( \frac{x-b}{2\sqrt{Dt}} \right) - \operatorname{erfc} \left( \frac{x-a}{2\sqrt{Dt}} \right) \right],$$

where  $a < b$ . This is a solution of the diffusion equation (you do not need to show this). What is  $c(x, 0)$ ?

3. The objective of this exercise is to address the following question: with point-source diffusion, if you are located somewhere on the  $x$  axis, how long do you have to wait until the maximum concentration is reached?

a) For a fixed  $x > 0$ , sketch the point-source solution as a function of  $t$ .

b) For a fixed  $x > 0$ , show that the point-source solution reaches its maximum value when  $t = t_M$ , where  $t_M = x^2 / (2D)$ .

c) Suppose  $x = 1$  m. Compare the value of  $t_M$  in air and water (use the  $D$  values from Exercise 1 and your answer should be measured in days).

4. The Takahashi model for a cell cycle results in the equation

$$\frac{\partial n}{\partial t} + \nu_0 \frac{\partial n}{\partial \alpha} = d_0 \frac{\partial^2 n}{\partial \alpha^2},$$

where  $\nu_0$  and  $d_0$  are positive constants. This is equation (60), page 466, in the text. Also, note that this is an example of what is called a drift-diffusion equation.

a) Show that the following is a solution of the above equation.

$$n(\alpha, t) = \frac{1}{2\sqrt{\pi d_0 t}} \exp \left[ -\frac{(\alpha - \nu_0 t)^2}{4d_0 t} \right]$$

b) Sketch  $n$  as a function of  $\alpha$  (assume  $t > 0$ ). Fixing  $t > 0$ , find the value of  $\alpha$  where  $n$  reaches a maximum and determine its value there. Also, state what happens to  $n$  as  $\alpha \rightarrow \infty$ . What if  $\alpha \rightarrow -\infty$ ?

c) Describe the differences between the solution in part (a) and the point-source solution for the diffusion equation. Your comments should be based on your results from part (b) and the corresponding results for the point-source solution.