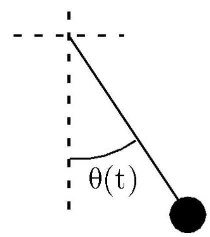
HW 4
Due Oct 22

1. The problem for a damped pendulum is

$$\theta' = v$$
$$v' = -\kappa v - \sin\theta$$

Physically θ corresponds to the angular displacement of the pendulum (see figure below), and v is the angular velocity. The constant κ is positive.
a) There are two steady-state positions. What are they?
b) Determine which steady-state is unstable and which is asymptotically stable.



2. text pg 261, problem 17 (note correction on pg xxxvii) with the following modifications:
a) same as text
b) find the four steady-states
c) same as text
d) One of the steady states has both N_1 and N_2 nonzero. Show that it is asymptotically stable.

3. text pg 363, problem 6 (note correction on pg xxxix) with the following modifications:
a) same as text
b) find the three steady-states
c) Of the two nonzero steady-states, one is asymptotically stable if $\mu < 1$ and unstable if $\mu > 1$, while the other is unstable if $\mu < 1$, asymptotically stable for $1 < \mu < 2$ and unstable if $2 < \mu$. Identify which is which (and provide the necessary math to justify your conclusion).
d) Is there a Hopf bifurcation at $\mu = 1$ or at $\mu = 2$?

Extra Credit: The paper "When Zombies Attack!: Mathematical Modelling of an Outbreak of Zombie Infection" contains a dynamical system for zombification (in Section 2. Basic Model).
a) Determine the four reactions that produce these equations (assume $\Pi = 0$ when doing this). Also, explain what each reaction means physically.
b) There is at least one unreasonable assumption in their model. What is it and how do the equations change? Note your correction(s) should not be one they consider (latent infection, quarantine, or a cure for zombie-ism).