Solutions to HW3

1. $y' = e^{-y} \Rightarrow e^{y} dy = dt \Rightarrow e^{y} = t + c \Rightarrow y = \ln(t+c)$

$y(0) = 0 \Rightarrow 0 = \ln c \Rightarrow c = 1$ or $y = \ln(1+t)$

2. $y' = 1 - \frac{y}{1+y} = \frac{1}{1+y} \Rightarrow (1+y)\,dy = dt$

$\Rightarrow y + \frac{1}{2}y^2 = t + c$

$y(0) = -2 \Rightarrow -2 + \frac{1}{2}(-2)^2 = c \Rightarrow c = 0$

$y + \frac{1}{2}y^2 = t$

$\Rightarrow$

$y^2 + 2y - 2t = 0$

$\Rightarrow$

$y = \frac{1}{2}\left(-2 \pm \sqrt{4 - 4\cdot(-2t)}\right)$

$= -1 \pm \sqrt{1+2t}$ $y(0) = -2 \Rightarrow$ minus sign

$= -1 - \sqrt{1+2t}$

3. a) $y' = 0 \Rightarrow y = 2$ or $y = 3$

$f(y) = (y-2)(y-3) \Rightarrow f' = y - 3 + y - 2 = 2y - 5$

$y = 2$: $f'(2) = -1 < 0 \Rightarrow$ asy. stable

$y = 3$: $f'(3) = 1 > 0 \Rightarrow$ unstable

b) $y' = 0 \Rightarrow y = 1$

$f(y) = \ln y \Rightarrow f'(y) = \frac{1}{y} \Rightarrow f'(1) = 1 > 0 \Rightarrow$ unstable

4. Note that x & y increase if $z = 0$, and decrease if $z \neq 0$; also, z decreases if x & y are zero. The conclusion is that x & y are prey & z is the predator. Also, x & y increase with y which indicates x is the male and y is the female.

$$x' = ay - bxz$$

birth rate prop. to female prey

death rate prop. to number of predators & to number of males (prey)

note same rate for males & females

$$y' = ay - cyz$$

death rate prop. to number of predators & number of female prey

$$z' = -dz + e(x+y)z$$

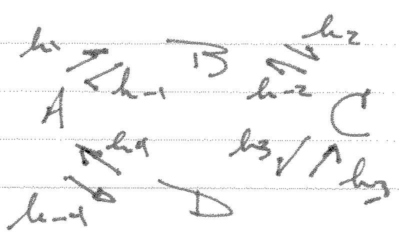
death rate prop. to number of predators

birth rate prop. to total number of prey (so, no difference between eating males or females)

5. a) $A + 2B \xrightarrow{k} C + 2D$

$\Rightarrow$

$$A' = -kAB^2 \qquad B' = -2kAB^2$$
$$C' = kAB^2 \qquad D' = 2kAB^2$$

b) 

$$A' = -k_1 A + k_{-1} B - k_{-4} A + k_4 D$$

$$B' = k_1 A - k_{-1} B - k_2 B + k_{-2} C$$

$$C' = k_2 B - k_{-2} C - k_3 C + k_{-3} D$$

$$D' = -k_4 D + k_{-4} A + k_3 C - k_{-3} D$$