HW 3
Due Oct 10

1. Find the solution of $y' = e^{-y}$ which satisfies $y(0) = 0$.

2. Find the solution(s) of $y' = 1 - \frac{y}{1+y}$ which satisfies $y(0) = -2$.

3. The following equations are of the form $y' = f(y)$. Find the steady-state(s) and determine if they are unstable or asymptotically stable. You can assume that $y > 0$, and you do not need to solve the equation.
a) $y' = (y-2)(y-3)$
b) $y' = \ln y$

4. Some predator-prey models separate the prey into males and females. One such model is:

$$\begin{aligned} x' &= ay - bxz \\ y' &= ay - cyz \\ z' &= -dz + e(x+y)z \end{aligned}$$

Determine which variable is used for the male prey, the female prey, and the predators. Also, explain the biological significance of each term on the right-hand side of these equations (include an explanation of why the terms of the male and female equations are the same or different).

5. a) A reaction is shown in Figure 1. Label each species in the reaction, write down the corresponding reaction and from this derive the dynamical system for this reaction. Note that to label them you are simply attaching a letter to each species (e.g., A, B, etc or X, Y, etc). You do not have to identify them physically (although, if you are interested, this is a reaction in which methane combines with oxygen to form carbon dioxide and water).
b) An ion channel is a gated pore that allows K^+ and Na^+ ions to cross a membrane. One proposed model for this is shown in Figure 2. There are four states in this model. Label each state, write down the corresponding reactions, and from this derive the dynamical system for this model.

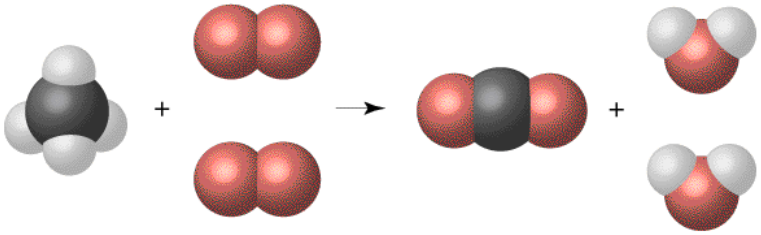

Figure 1: Reaction for Exercise 5(a).

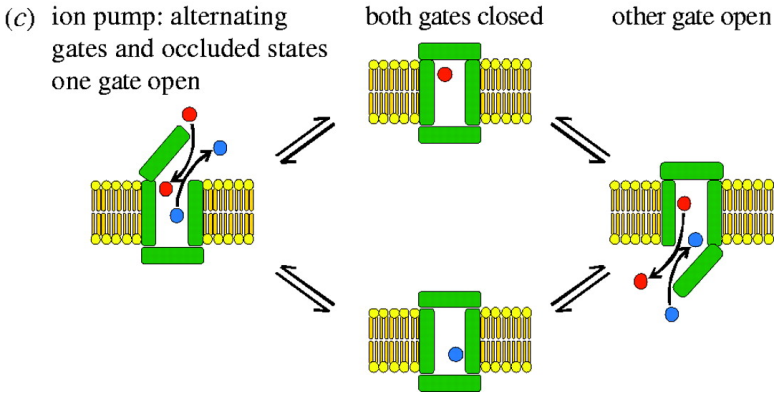

Figure 2: Reaction for Exercise 5(b). Note red designates a K^+ ion and blue is a Na^+ ion. (Ref: "Peering into an ATPase ion pump with single-channel recordings," Gadsby, et al, Phil. Trans. R. Soc. B, 27 Jan 2009, vol. 364, 229-238.)