2. a) In diffusion, suppose $D_1 = 1\frac{cm^2}{sec}$ and $D_2 = 2\frac{cm^2}{sec}$. Explain the difference between the random walks in the two situations.

The key here is the formula $D = \frac{\Delta x^2}{2\Delta t}$.
So, for a given time step Δt, having D increase by a factor of 2 means the distance Δx traveled in that time step is $\sqrt{2}$ larger.

b) The following system is known to have a Turing instability

$$\partial_t u = D_1 \partial_x^2 u + u^4 - v$$
$$\partial_t v = D_2 \partial_x^2 v + u - v^3$$

Which one is the activator? Which one has the bigger diffusion coefficient? Make sure to provide a clear explanation in each case.

The activator has an autocatalytic term – which for the above equations means u is the activator. It is also the one that is slowly diffusing – so it has the smaller diffusion coefficient.

3. Two species interact as follows

$$A \to 2A + B \qquad r_1 = k_1 A$$
$$B \to A. \qquad r_2 = k_2 B$$

It is also known that species A moves away from higher B densities, while B moves towards higher A densities. What are the resulting partial differential equations that comes from these assumptions?

Q's: $A' = -r_1 + 2r_1 + r_2 = r_1 + r_2 = k_1 A + k_2 B$

$B' = r_1 - r_2 = k_1 A - k_2 B$

J's: for A $\quad J = -\alpha \frac{\partial B}{\partial x}$

away from increasing B

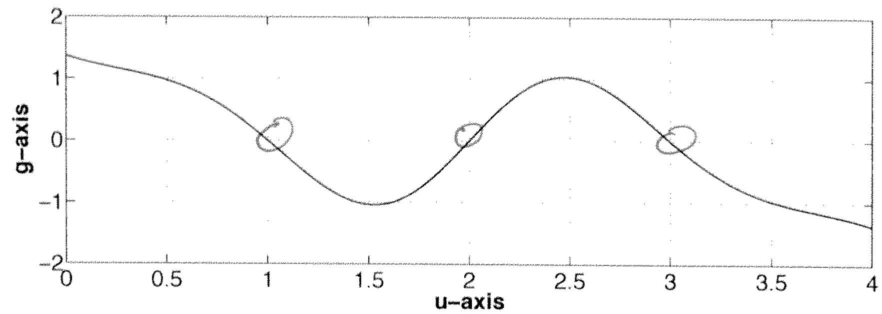
for B $\quad J = \beta \frac{\partial A}{\partial x}$

pos. since goes towards larger A

Resulting equations

$$\frac{\partial}{\partial t} A = \alpha \frac{\partial^2 B}{\partial x^2} + k_1 A + k_2 B$$
$$\partial_t B = -\beta \partial_x^2 A + k_1 A - k_2 B$$

4. Consider the reaction-diffusion equation $\partial_t u = D\partial_x^2 u + g(u)$, where $g(u)$ is shown below.
a) Find the constant steady-states and determine their stability.



steady-states: $g(u) = 0 \Rightarrow u = 1,\ u = 2$ or $u = 3$

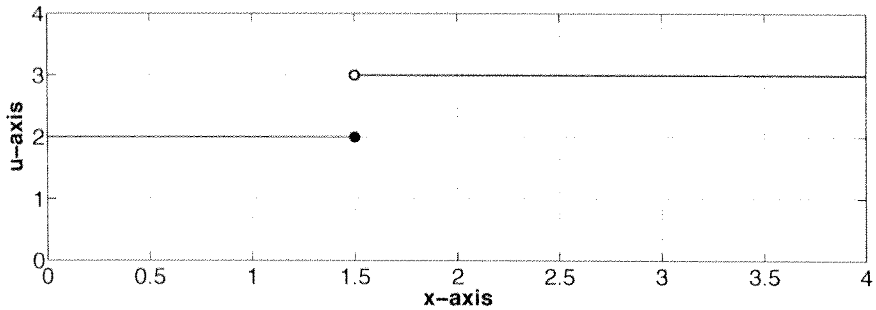
stability: ~~g'~~

$u = 1$: $g'(1) < 0$ (from graph) $\Rightarrow$ stable

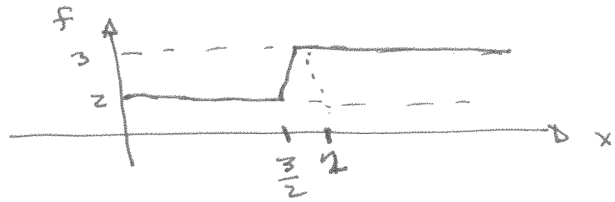
$u = 2$: $g'(2) > 0 \Rightarrow$ unstable

$u = 3$: $g'(3) < 0 \Rightarrow$ stable

b) The graph of u at $t = \infty$ is given below. Give an example of an initial condition $u(x, 0) = f(x)$ that would produce this result.



It's assumed $f(x)$ is continuous — and so, possibilities ar



$$f = \begin{cases} \frac{3}{2} & \text{if } x \le \frac{3}{2} \\ \text{linear} & \text{for } \frac{3}{2} < x < 2 \\ 3 & \text{for } x \ge 2 \end{cases}$$

or

reason: $f = 2$ for $0 \le x \le \frac{3}{2}$ $\Rightarrow$ sol (u) stays at $u = 2$ in this interval

$2 < f \le 3$ for $x > \frac{3}{2}$ $\Rightarrow$ $u \to 3$ as $\cancel{t} \to \infty$ in this interval