

CSUMS Project: Numerical Simulations of Porous Medium Equation

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The main goal of this project is to simulate the porous medium equation $u_t = (u^m)_{xx}$, $m > 1$ reliably and accurately. This equation arises in various applications such as ground-water infiltration, heat radiation in plasma, population dynamics. The solutions manifest many interesting features. The effort of this project includes identifying, possibly improving, and implementing numerical methods to solve such equation. One promising choice for the robust numerical algorithms is discontinuous Galerkin (DG) method. Other possible directions of this project include: applying DG method to other interesting problems, or to explore other numerical methods to solve porous medium equation.

References:

1. Discontinuous Galerkin methods: general approach and stability, by C.-W. Shu, to appear in *Advanced Courses in Mathematics – CRM Barcelona*, Birkhauser, Springer.

The paper contains general description of DG methods for solving time-dependent differential equations. Detailed formulation of the methods are given through several one dimensional examples.

2. A discontinuous Galerkin finite element method for time dependent partial differential equations with higher order derivative, by Y. Cheng and C.-W Shu, *Mathematics of Computation*, v77 (2008), pp.699-730.

This paper provides a good candidate of the numerical algorithms for this CSUMS project in order to simulate the porous medium equation.

3. Numerical simulation for porous medium equation by local discontinuous Galerkin finite element method, by Q. Zhang and Z.-L. Wu, *Journal of Scientific Computing*, v38 (2009), pp127-148.

This paper reports a recent work on using one type of discontinuous Galerkin method to solve porous medium equation. The method we want to work on is different, and is in general more compact. This paper provides a great reference to our project in terms of understanding both the equation and the algorithms.

4. Regularity properties of flows through porous media, by D.G. Aronson, *SIAM Journal on Applied Mathematics*, v17 (1969), p461-467

The first page of the paper outlines how the equation is derived from the basic physical laws.

5. The Porous Medium Equation, Mathematical Theory, by Juan Luis Vazquez (book preview) <http://books.google.com/books?id=bCfk0pKwJOUC>

Quite much of the background information regarding the porous medium equation can be found in this book preview. For instance, the features of such equations are outlined on page 4, and starting from page 19, several examples of applications are discussed.