Numerical Computing F25
Exam 2 Name: ______________________________

There are 8 questions. No calculators or notes are to be used. Make sure to show your work, any answer without supporting work will receive no credit.

1. In this problem $x_1 = -1$, $x_2 = 1$, $x_3 = 3$, $x_4 = 4$, and $p_3(x) = \ell_1(x) - \ell_3(x) + \ell_4(x)$, for $-1 \leq x \leq 4$.

(a) What data points (x_i, y_i) were used to produce $p_3(x)$?

(b) Rewrite $p_3(x)$ using the modified Lagrange interpolation formula.

2. a) Suppose piecewise linear interpolation is used to interpolate the data points: $(0,1)$, $(1,-1)$, $(2,3)$, $(3,-1)$, $(4,0)$, $(5,1)$. If $g(x)$ is the interpolation function, evaluate $g(2.5)$.

b) Runge's function was used to demonstrate why a global interpolation polynomial $p_n(x)$ can produce a very poor approximation for larger values of n. What happens with $p_n(x)$ that is the reason for making this conclusion?

3. In this problem $x_1 = -2$, $x_2 = 0$, $x_3 = 2$, and

$$s(x) = B_0(x) + 2B_1(x) - B_2(x) + B_3(x) - B_4(x),$$

for $-2 \le x \le 2$.

(a) What is x_0 and what is x_4?

(b) What data points (x_i, y_i) were used to produce this cubic spline?

(c) Is this a natural cubic spline? Why?

4. The function $f(x) = 3x^4 + 5$ is going to be approximated using piecewise linear interpolation at equally spaced nodes for $1 \leq x \leq 6$. How many data points are needed to guarantee that the value of the interpolation function is correct to eight significant digits at every point in the interval?

5. Given the values of $f(x)$ in the table below, the objective is to calculate $F(x) = \int_0^x f(s)ds$. You are to use as many of the data points as possible when answering each question. Note that part (c) of this question is on the next page.

x	0	1	2	3	4
f	1	0	-2	-1	2

(a) Use the trapezoidal (or composite trapezoidal) rule to find the value of $F(x)$ at $x = 1, 2, 3, 4$.

(b) What x's can $F(x)$ be evaluated at using the midpoint (or composite midpoint) rule? What is the value of $F(x)$ at each of these points?

(c) Use Romberg integration with the composite Simpson's rule to calculate $F(4)$.

x	0	1	2	3	4
f	1	0	-2	-1	2

6. a) What approximation of $f(x)$ is used to obtain the midpoint rule? You do not need to derive the approximation, only state what it is (along with a sketch illustrating the approximation).

b) The approximation of $f(x)$ used for the midpoint rule is not as good as the one used for the trapezoidal rule, yet it produces a better approximation of the integral. Using your sketch from part (a), explain why.

7. a) The composite Simpson's rule requires there to be an even number of subintervals. What is the reason?

b) When using 20 subintervals the relative iterative error using the composite Simpson's rule is 10^{-3}. What is the expected relative iterative error when using 40 subintervals?

8. Suppose the integration rule

$$\int_a^b f(x)dx \approx wf(a+z) + wf(a+2z)$$

is used. Find w and z that maximize the precision.

Worksheet