Numerical Computing F'25
Exam 1 Name: _____

There are 7 questions. Make sure to show your work (any answer without supporting work will receive no credit). Do not use a crib sheet, calculator, phone, etc.

1. Assume that double precision is being used. Provide either a sentence or diagram that explains how you arrived at the answer.

a) What is the spacing of (i.e., how far apart are) the floating point numbers in the interval $20 \leq x \leq 30$?

b) What value will be computed for: $1 + \pi\varepsilon$?

2. This problem concerns computing the value of

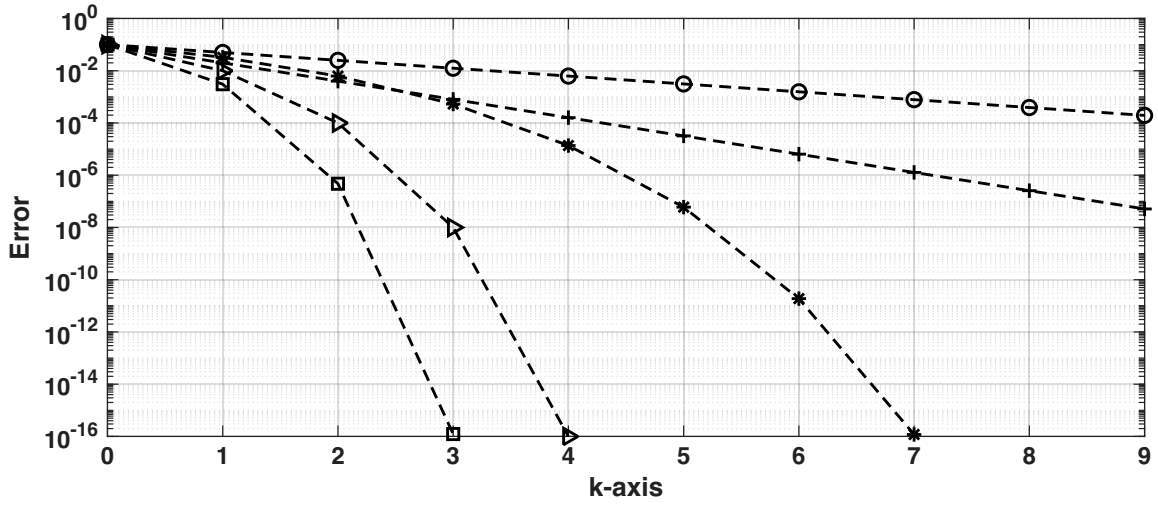
$$\sum_{k=0}^{1000} \frac{e^k - 1}{e^k + 1}$$

a) Explain why computing this sum, as written, will fail.

b) How can the expression be rewritten so it can be computed? This requires a sentence or two explaining why this works.

3. For the secant method, given x_k and x_{k-1}, there is a formula to find x_{k+1}. What approximation is used to obtain this formula? A sketch should be used when answering this question. Use this approximation to derive the formula for x_{k+1}.

4. The graph below gives the value of the error $|x_k - \bar{x}|$ when solving $f(x) = 0$ using five different iteration methods.



a) Which curve corresponds to the bisection method? Why?

b) Which curve corresponds to Newton's method? Why?

5. Find the Doolittle factorization of

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -1 \end{pmatrix}.$$

6. Below is a matrix and its inverse (assume that $a > 2$)

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ a & 0 & 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ -a & 0 & 1 \end{pmatrix}$$

a) Find $\kappa_\infty(\mathbf{A})$.

b) If the residual $\mathbf{r} = (0, 2, 1)^T$, what is the error $\mathbf{e}$?

c) In solving $\mathbf{Ax} = \mathbf{b}$, if $a = 10^5$ then the computed solution is likely correct to no more than how many digits? Why?

7. a) Fill in the blanks:

Assuming $\mathbf{A}$ is symmetric and positive definite, solving $\mathbf{Ax} = \mathbf{b}$ using a Cholesky factorization takes approximately _____ the flops needed when using a LU factorization because you do not need to compute _____ .

b) Find the Cholesky factorization of $\mathbf{A}$ (you do not need to prove it is positive definite).

$$\mathbf{A} = \begin{pmatrix} 4 & 0 & 2 \\ 0 & 9 & 0 \\ 2 & 0 & 10 \end{pmatrix}$$

Worksheet