Type	**Difference Approximation**	**Truncation Term**
Forward	$y'(t_j) \approx \frac{y(t_{j+1})-y(t_j)}{k}$	$\tau_j = -\frac{1}{2}ky''(\eta_j)$
Backward	$y'(t_j) \approx \frac{y(t_j)-y(t_{j-1})}{k}$	$\tau_j = \frac{1}{2}ky''(\eta_j)$
Centered	$y'(t_j) \approx \frac{y(t_{j+1})-y(t_{j-1})}{2k}$	$\tau_j = -\frac{1}{6}k^2y'''(\eta_j)$
One-sided	$y'(t_j) \approx \frac{-y(t_{j+2})+4y(t_{j+1})-3y(t_j)}{2k}$	$\tau_j = \frac{1}{3}k^2y'''(\eta_j)$
One-sided	$y'(t_j) \approx \frac{3y(t_j)-4y(t_{j-1})+y(t_{j-2})}{2k}$	$\tau_j = \frac{1}{3}k^2y'''(\eta_j)$
Centered	$y''(t_j) \approx \frac{y(t_{j+1})-2y(t_j)+y(t_{j-1})}{k^2}$	$\tau_j = -\frac{1}{12}k^2y''''(\eta_j)$

Table 1: Numerical differentiation formulas. These formulas assume equally spaced points with step size $k = t_{j+1} - t_j$, and the point η_j is located between the left- and rightmost points used in the formula.

Rule	**Integration Formula**
Right Box	$\int_{t_j}^{t_{j+1}} f(x)dx = kf(t_{j+1}) + O(k^2)$
Left Box	$\int_{t_j}^{t_{i+1}} f(x)dx = kf(t_j) + O(k^2)$
Midpoint	$\int_{t_{j-1}}^{t_{j+1}} f(x)dx = 2kf(t_j) + \frac{k^3}{3}f''(\eta_i)$
Trapezoidal	$\int_{t_j}^{t_{j+1}} f(x)dx = \frac{k}{2}(f(t_j) + f(t_{j+1})) - \frac{k^3}{12}f''(\eta_j)$
Simpson	$\int_{t_{j-1}}^{t_{j+1}} f(x)dx = \frac{k}{3}(f(t_{j+1}) + 4f(t_j) + f(t_{j-1})) - \frac{k^5}{90}f''''(\eta_j)$

Table 2: Numerical integration formulas. The points $t_1, t_2, t_3, \ldots$ are equally spaced with step size $k = t_{j+1} - t_j$. The point η_j is located within the interval of integration.

Methods for solving the differential equation

$$\frac{dy}{dt} = f(t, y)$$

Method	**Difference Equation**	τ_j	**Properties**
Euler	$y_{j+1} = y_j + kf_j$	$O(k)$	Explicit; Conditionally A-stable
Backward Euler	$y_{j+1} = y_j + kf_{j+1}$	$O(k)$	Implicit; A-stable
Trapezoidal	$y_{j+1} = y_j + \frac{k}{2}(f_j + f_{j+1})$	$O(k^2)$	Implicit; A-stable
Heun (RK2)	$y_{j+1} = y_j + \frac{1}{2}(k_1 + k_2)$ where $k_1 = kf_j, \qquad k_2 = kf(t_{j+1}, y_j + k_1)$	$O(k^2)$	Explicit; Conditionally A-stable
Classic Runge–Kutta (RK4)	$y_{j+1} = y_j + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ where $k_1 = kf_j, \quad k_2 = kf(t_j + \frac{k}{2}, y_j + \frac{1}{2}k_1),$ $k_3 = kf(t_j + \frac{k}{2}, y_j + \frac{1}{2}k_2), \quad k_4 = kf(t_{j+1}, y_j + k_3)$	$O(k^4)$	Explicit; Conditionally A-stable

Table 3: Finite difference methods for solving an IVP. The points $t_1, t_2, t_3, \ldots$ are equally spaced with step size $k = t_{j+1} - t_j$. Also, $f_j = f(t_j, y_j)$, $f_{j+1} = f(t_{j+1}, y_{j+1})$ and τ_j is the truncation error for the method.