Summary: Polynomial Interpolation (Chapter 5)

Interpolation Points: $(x_1, y_1), (x_2, y_2), \cdots, (x_{n+1}, y_{n+1})$, where $x_1 < x_2 < \cdots < x_{n+1}$

Global Polynomial Interpolation

$$p_n(x) = \ell(x) \sum_{i=1}^{n+1} \frac{w_i y_i}{x - x_i} \tag{5.8}$$

where

$$\ell(x) = (x - x_1)(x - x_2) \cdots (x - x_{n+1}) = \prod_{j=1}^{n+1} (x - x_j)$$

and

$$w_i = \frac{1}{(x_i - x_1)(x_i - x_2) \cdots (x_i - x_{i-1})(x_i - x_{i+1}) \cdots (x_i - x_{n+1})} = \left(\prod_{\substack{j=1 \\ j \neq i}}^{n+1} (x_i - x_j) \right)^{-1}$$

Chebyshev interpolation: $x_i = \frac{1}{2}[a + b + (b - a) z_i]$, where $z_i = \cos\!\left(\frac{2i-1}{2(n+1)}\pi\right)$

Piecewise Linear Interpolation

$$g(x) = \sum_{i=1}^{n+1} y_i G_i(x) \tag{5.15}$$

where

$$G_i(x) = \begin{cases} 0 & \text{if } \ x \leq x_{i-1}, \\ \dfrac{x - x_{i-1}}{x_i - x_{i-1}} & \text{if } \ x_{i-1} \leq x \leq x_i, \\ \dfrac{x - x_{i+1}}{x_i - x_{i+1}} & \text{if } \ x_i \leq x \leq x_{i+1}, \\ 0 & \text{if } \ x_{i+1} \leq x \end{cases}$$

Equally spaced nodes:

$$G_i(x) = G\left(\frac{x - x_i}{h} \right)$$

and

$$G(x) = \begin{cases} 1 - |x| & \text{if } \ |x| \leq 1, \\ 0 & \text{if } \ 1 \leq |x| \end{cases}$$

Piecewise Cubic Interpolation (equally spaced)

$$s(x) = \sum_{i=0}^{n+2} a_i B_i(x) \tag{5.27}$$

where

$$B_i(x) = B\left(\frac{x - x_i}{h}\right) \tag{5.23}$$

and

$$B(x) = \begin{cases} \frac{2}{3} - x^2\left(1 - \frac{1}{2}|x|\right) & \text{if } \ |x| \leq 1, \\ \frac{1}{6}(2 - |x|)^3 & \text{if } \ 1 \leq |x| \leq 2, \\ 0 & \text{if } \ 2 \leq |x| \end{cases}$$

Natural Spline: $s''(x_1) = 0$ and $s''(x_{n+1}) = 0$
Clamped Spline: $s'(x_1) = y_1'$ and $s'(x_{n+1}) = y_{n+1}'$
Not-a-Knot Spline (for $n > 2$): $s_1'''(x_2) = s_2'''(x_2)$ and $s_{n-1}'''(x_n) = s_n'''(x_n)$

	x_{i-1}	x_i	x_{i+1}	x_j for $j \neq i, i \pm 1$
B_i	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	0
B_i'	$\frac{1}{2h}$	0	$-\frac{1}{2h}$	0
B_i''	$1/h^2$	$-2/h^2$	$1/h^2$	0

Table 1: Values of $B_i(x)$ at the nodes.

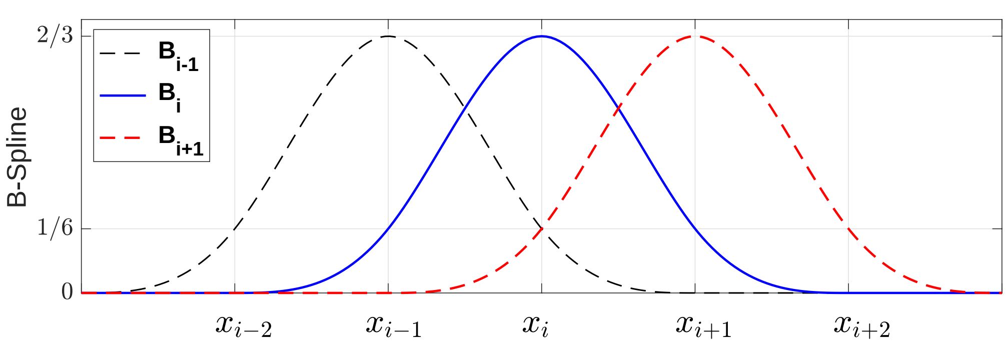

Figure 1: Plot of the cubic B-splines $B_{i-1}(x)$, $B_i(x)$, and $B_{i+1}(x)$.