Summary: Composite Integration (Chapter 6)

Midpoint

$$I_M = h\left[f\Big(a+\frac{1}{2}h\Big) + f\Big(a+h\Big) + f\Big(a+\frac{3}{2}h\Big) + \cdots + f\Big(a+(n-\frac{1}{2})h\Big)\right] \tag{6.9}$$

where $E_M = \frac{b-a}{24}h^2 f''(\eta)$. Romberg: $\int_a^b f(x)dx = \frac{1}{3}\big[4I_M(2n) - I_M(n)\big] + O(h^3)$

Trapezoidal

$$I_T = h\left(\frac{1}{2}f_1 + f_2 + f_3 + \cdots + f_n + \frac{1}{2}f_{n+1}\right) \tag{6.15}$$

where $E_T = -\frac{b-a}{12}h^2 f''(\eta)$. Romberg: $\int_a^b f(x)dx = \frac{1}{3}\big[4I_T(2n) - I_T(n)\big] + O(h^3)$

Simpson

$$I_S = \frac{h}{3}(f_1 + 4f_2 + 2f_3 + 4f_4 + 2f_5 + \cdots + 4f_n + f_{n+1}) \tag{6.22}$$

where $E_S = -\frac{b-a}{180}h^4 f''''(\eta)$. Romberg: $\int_a^b f(x)dx = \frac{1}{15}\big[16I_S(2n) - I_S(n)\big] + O(h^6)$

Hermite

$$I_H = I_T + \frac{1}{12}h^2\big(f_1' - f_{n+1}'\big) \tag{6.28}$$

where $E_H = \frac{b-a}{720}h^4 f''''(\eta)$. Romberg: $\int_a^b f(x)dx = \frac{1}{15}\big[16I_H(2n) - I_H(n)\big] + O(h^6)$

Discrete Data (via Cubic Splines)

$$\int_a^b f(x)dx \approx \frac{1}{3}I_T + \frac{2}{3}\sum_{i=1}^{n} h_i s\Big(x_i + \frac{1}{2}h_i\Big) \tag{6.40}$$

where $h_i = x_{i+1} - x_i$.

Gaussian

$$I_G = w_1 f(z_1) + w_2 f(z_2) + \cdots + w_m f(z_m) \tag{6.42}$$

where $|E_G| \le \frac{\alpha}{\sqrt{m}} R^{2m} ||f^{(2m)}||_\infty$ for $R = (b-a)e/(8m)$ and $\alpha = (b-a)\sqrt{\pi}/4$

k	$f(x)$	$\int_a^b f(x)dx$
0	1	ℓ
1	x	ℓx_m
2	x^2	$\ell\left(x_m^2 + \frac{1}{12}\ell^2\right)$
3	x^3	$\ell x_m\left(x_m^2 + \frac{1}{4}\ell^2\right)$
4	x^4	$\ell\left(x_m^4 + \frac{1}{2}\ell^2 x_m^2 + \frac{1}{80}\ell^4\right)$
5	x^5	$\ell x_m\left(x_m^4 + \frac{5}{6}\ell^2 x_m^2 + \frac{1}{16}\ell^4\right)$

Table 1: Values for $\int_a^b x^k dx$, where $\ell = b - a$ and $x_m = (b+a)/2$.